



# GYROSCOPIC WHEEL

AHS STEM Activity: **College Level**

Jan 2018

## Why does a helicopter need a tail rotor?

Newton's third law states that every action has an equal and opposite reaction. When a helicopter's main rotor (propeller) spins, the helicopter body will follow this law, and try to spin in the opposite direction! This is what we call torque.

To stop this from happening, we need to find a way to conserve the momentum of the main rotor. This is why we use a tail rotor, which works to fight against torque. This is why, in many Hollywood movies, when the tail is destroyed, the helicopter spirals out of control.<sup>1</sup>

## The Demo

Someone stands on a rotating platform. When they spin the bicycle wheel and turn it sideways, they also spins around. What's going on?<sup>2</sup>

## Quick Physics

Conservation of Angular Momentum means that the person turns in the opposite direction from the spinning bicycle wheel.<sup>2</sup>



Rotating platform and bicycle wheel

## Required:

- Rotating platform (or rotating chair)
- Bicycle wheel with handles
- A partner



## The Details

This illustrates an important conservation law of physics: the conservation of angular momentum or turning motion. Anything that is turning has angular momentum. This is similar to inertia, only in rotational motion. If a wheel is spinning in one direction, it wants to keep turning in that same direction. If it is not turning, it tries to stay still.

To show this, someone stands on a level platform that can spin around. If that person gets on carefully, so they're not turning, no matter how much that person twists and dances, they won't be able to spin all the way around. Next, someone else can push the person holding the spinning wheel so that person starts spinning. Now, no matter how hard that person tries, they won't be able to stop spinning. This is conservation of angular momentum.

In the bicycle wheel gyroscope demonstration, there are two possible turning motions. The platform the person stands on turns like a wheel on its side. Call this horizontal turning motion. The other turning motion is the bicycle wheel the person is holding. Initially it is vertical, as a bicycle wheel normally stands. Call this vertical turning motion.

To begin with, someone spins the bicycle wheel. There is vertical turning motion in the bicycle wheel, but no horizontal turning motion of the platform. Then the person tilts the bicycle wheel to the side, so now there is horizontal turning motion where there wasn't any before. Conservation of angular momentum tries to fix this. The platform starts spinning in the direction opposite that of the bicycle wheel. If you add the backward turning of the platform and the forward turning of the bike wheel together, you get zero. So horizontal motion is the same as when we started: equaling zero or no motion. When the bike wheel is tilted back vertically, the platform stops spinning, as in the beginning.

Rocket ships are stabilized this way. They have little wheels spinning inside called a gyroscope. When the rocket ship turns, the spinning wheels turn the other way and bring the rocket back on course. Football players also put a spin on the football to improve the accuracy of their passes.<sup>2</sup>

## Sources

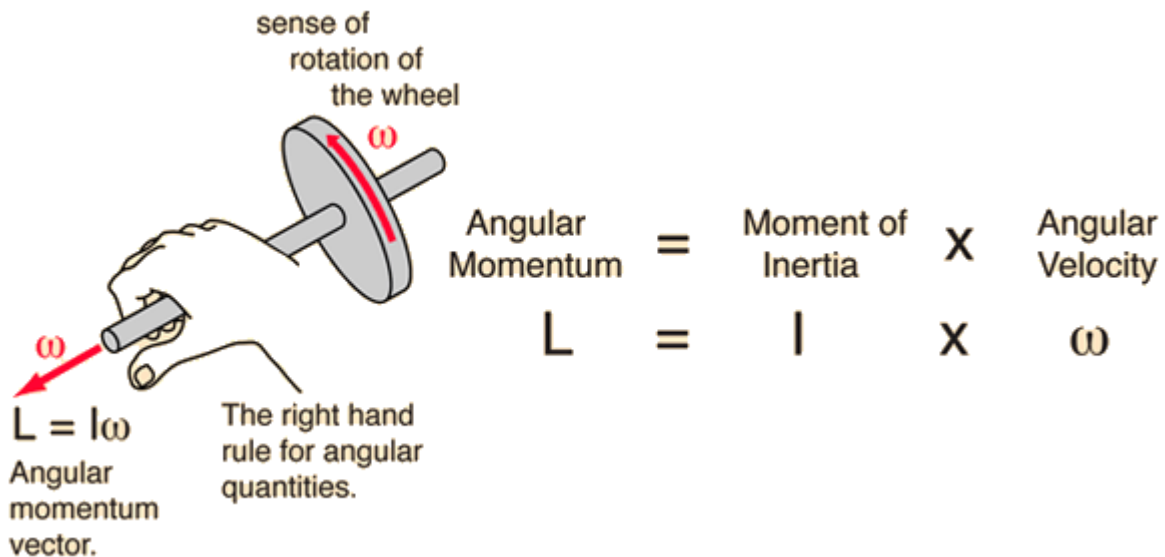
<sup>1</sup>Smithsonian National Air and Space Museum: <http://howthingsfly.si.edu/ask-an-explainer/why-do-helicopters-need-tail-rotor-and-what-torque>

<sup>2</sup>The Wonders of Physics - Traveling Outreach Program: <https://wonders.physics.wisc.edu/the-bicycle-wheel-gyroscope/>

# Notes (1/3)

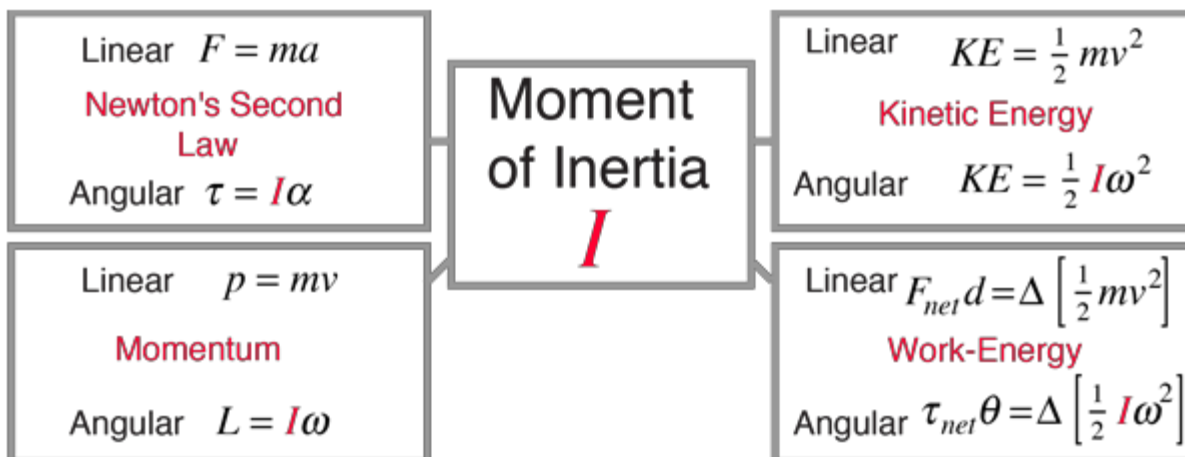
## Angular Momentum

The angular momentum of a rigid object is defined as the product of the moment of inertia and the angular velocity. It is subject to the fundamental constraints of the conservation of angular momentum principle if there is no external torque on the object. Angular momentum is a vector quantity. It is derivable from the expression for the angular momentum of a particle.<sup>3</sup>



## Moment of Inertia

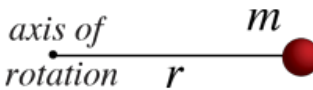
Moment of inertia is the name given to rotational inertia, the rotational analog of mass for linear motion. It appears in the relationships for the dynamics of rotational motion. The moment of inertia must be specified with respect to a chosen axis of rotation. For a point mass the moment of inertia is just the mass times the square of perpendicular distance to the rotation axis,  $I = mr^2$ . That point mass relationship becomes the basis for all other moments of inertia since any object can be built up from a collection of point masses.<sup>3</sup>



# Notes (2/3)

## Common Moments of Inertia

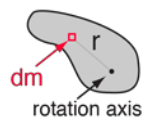
The moment of inertia of a point mass is defined as

$$I = mr^2$$


The diagram shows a horizontal line representing the axis of rotation. A red dot representing mass  $m$  is located at a distance  $r$  from the axis.

The moment of inertia of an ordinary object involves a continuous distribution of mass at a continually varying distance from any rotation axis, the calculation of moments of inertia generally involves calculus, the discipline of mathematics which can handle such continuous variables.<sup>3</sup>

The moment of inertia contribution by an infinitesimal mass element  $dm$  has the same form as the moment of inertia of a point mass. This kind of mass element is called a differential element of mass and its moment of inertia is given by



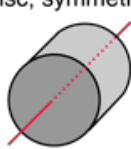

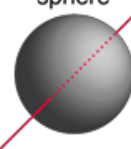
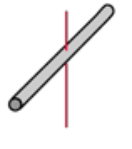
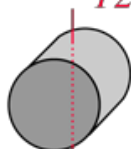
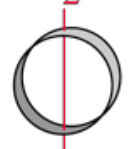
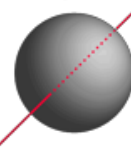
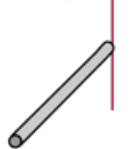
$$dI = r^2 dm$$

The "d" preceding any quantity denotes a vanishingly small or "differential" amount of it.

Note that the differential element of moment of inertia  $dI$  must always be defined with respect to a specific rotation axis. The sum over all these mass elements is called an integral over the mass.<sup>3</sup>

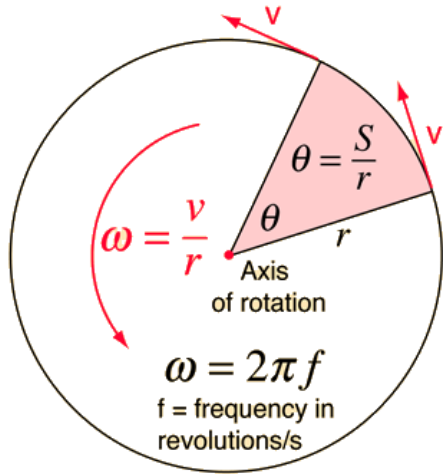
$$I = \int dI = \int_0^M r^2 dm$$

Common moments of inertia:

<p>Solid cylinder or disc, symmetry axis</p>  $I = \frac{1}{2} MR^2$	<p>Hoop about symmetry axis</p>  $I = MR^2$	<p>Solid sphere</p>  $I = \frac{2}{5} MR^2$	<p>Rod about center</p>  $I = \frac{1}{12} ML^2$
<p>Solid cylinder, central diameter</p>  $I = \frac{1}{4} MR^2 + \frac{1}{12} ML^2$	<p>Hoop about diameter</p>  $I = \frac{1}{2} MR^2$	<p>Thin spherical shell</p>  $I = \frac{2}{3} MR^2$	<p>Rod about end</p>  $I = \frac{1}{3} ML^2$

# Notes (3/3)

## Angular Velocity



For an object rotating about an axis, every point on the object has the same angular velocity. The tangential velocity of any point is proportional to its distance from the axis of rotation. Angular velocity has the units rad/s.<sup>3</sup>

$$v = \omega r \quad \text{or} \quad \omega = \frac{v}{r}$$

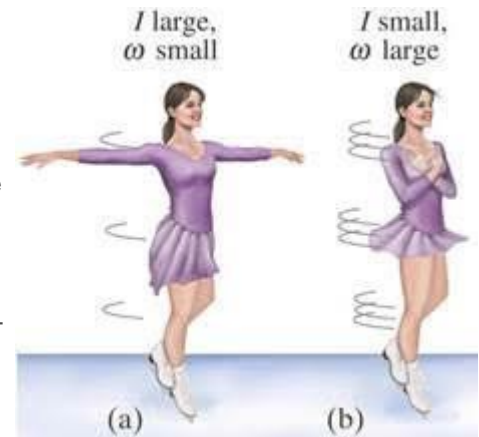
## Sources

<sup>3</sup>HyperPhysics: <http://hyperphysics.phy-astr.gsu.edu/hbase/amom.html#am>

## Examples

### Ice Skater

Angular momentum is conserved when no external torque is applied. Since there are no external moments acting on the skater, the angular momentum is the same in (a) and (b). By retracting her arms, the ice skater reduces her moment of inertia by reducing the radius. For angular momentum to stay the same, the angular velocity increases to compensate for the decrease in the moment of inertia.



### Helicopter

Because of conservation of angular momentum, when there are no external moments, the sum of the angular momentum of the helicopter will be zero. When the blades of the helicopter spin clockwise, the fuselage (body) of the helicopter will spin in the counterclockwise direction. A spinning tail rotor provides a clockwise moment to counterbalance the fuselage.



# Worksheet

## Problems

1. What is the angular momentum of a 0.25 kg mass rotating on the end of a piece of rope in a circle of radius 0.75m at an angular speed of 12.5 rad/s?
2. A figure skater rotates on ice at a rate of 3.5 rad/s with her arms extended horizontally. When she lowers her arms to her side, she speeds up to 7.5 rad/s. Find the ratio of her moment of inertia in the first case to that in the second case.
3. A disk has a mass of 3.5 kg and radius of 15 cm is rotating with an angular speed of 15 rev/s when a second non-rotating disk of 5.0 kg, mounted on the same shaft is dropped onto it. If the second disk has a diameter of 18 cm and a mass of 5.0 kg, what is the common final angular speed of the system?
4. A bowling ball has a mass of 5.5 kg and a radius of 12.0 cm. It is released so that it rolls down the alley at a rate of 12 rev/s. Find the magnitude of its angular momentum.
5. A student whose mass is 60 kg is standing on the edge of a circular merry-go-round facing inward. The merry-go-round has a mass of 100 kg and a radius of 2.0 m and spins at a rate of 2 rad/s. The student walks slowly from the outer edge toward the center and stops at a distance of 0.50m from the center. Calculate the magnitude of the angular velocity of the system.

## Answers

1. 1.8 kg.m<sup>2</sup>/s
2. 2.1
3. 9.9 rev/s
4. 0.76 kg.m<sup>2</sup>/s
5. 4.1 rad/s

## Sources

<sup>4</sup>[http://susanryan.weebly.com/uploads/2/2/3/1/22312194/day\\_58\\_homework.pdf](http://susanryan.weebly.com/uploads/2/2/3/1/22312194/day_58_homework.pdf)